Mesoscopic left-handed transmission lines in thermal Fock state

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The fluctuations of the current and voltage of the quantized unit cell equivalent circuit in loss-less mesoscopic left-handed transmission lines (LH TL)are deduced by thermal field dynamics (TFD) theory. And the fluctuations dependent of negative refractive index (NRI) of mesoscopic LH TL is discussed further in thermal Fock state. The results indicate that the quantum fluctuations show linear increasing dependent of NRI, while the frequency within the microwave frequency band and thermal photons show destructive dependent of NRI at some temperature. When the unit cell equivalent circuit operates at the rising temperature, the NRI is decreasing. The results demonstrates that the lower frequency and temperature, little thermal photons are more conducive to NRI of the mesoscopic LH TL, which is significant for the miniaturizing applications of LH TL.

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I. INTRODUCTION

The theoretical speculation of negative refractive index materials (NRM) proposed by V. Veselago[1] in 1968, in which several fundamental phenomena occurring in or in association with NRM were predicted, such as the negative Goos-Hänchen shift[2], amplification of evanescent waves[3], reversals of both Doppler shift and Cerenkov radiation[1], sub-wavelength focusing[4] and so on. Some typical approaches can be summarized as artificial structures such as metamaterials[5–7] and photonic crystals[8–10], chiral materials[11] and photonic resonant media[12, 13]. Although very exciting from a physics point of view, the artificial structures seem of little practical interest for engineering applications because of these resonant structures exhibiting high loss and narrow bandwidth consequently. Due to the weaknesses of resonant-type structures, three groups almost simultaneously in June 2002 introduced a transmission line (TL) approach of NRM: Eleftheriades et. al[14, 15], Oliner[16] and Caloz et. al.[17, 18]. LH TL initially the nonresonant-type one, is perhaps one of the most representative and potential candidates due to its low loss, broad operating frequency band, as well as planar configuration[19, 20], which is often related with easy fabrication for NRI applications in a suite of novel guided-wave[21], radiated-wave[22], and refracted-wave devices and structures[23, 24].

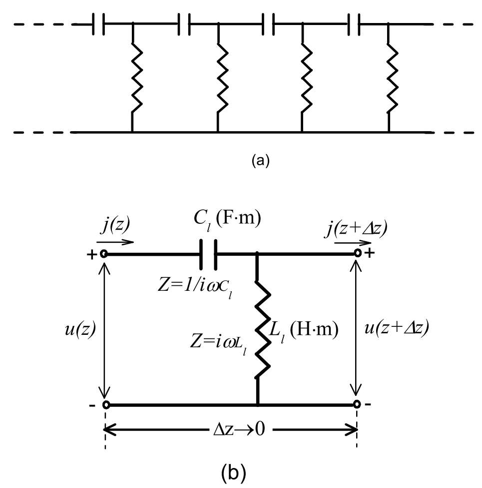

FIG. 1. (a)Equivalent circuit model for the hypothetical uniform Left-handed Transmission Line,(b)Unit cell equivalent circuit model for a hypothetical uniform Left-handed Transmission Line.

However, with the rapid development of nanotechnology and nanoelectronics [25], the integrated circuits and components have minimized towards atomic-scale dimensions[26, 27] in the last a few decades. When the scale of fabricated electric materials reached to a characteristic dimension, namely, Fermi wavelength, quan-

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tum mechanical properties of mesoscopic physics[28, 29] become important while the application of classical mechanics fails. The miniaturizing applications would be undoubtedly a persistent trend for LH TL, and quantum properties maybe its future challenge. So it's significant to investigate the quantum properties before the applications of LH TL approaching to nanometer scale.

Thus, from the point of against this challenge, this paper exploits the dependence of quantum properties of thermal noise, the quantum fluctuations of current and the frequency of LH TL on NRI. In Section 2 , we quantize the non-time-dependent Hamiltonian for the non-dissipative LH TL equivalent circuit via the analogy with harmonic oscillator. In Section 3, we derive the NRI for this LH TL circuit via the quantum fluctuations of current. And the quantum effect was then discussed by the mentioned results. Finally, in Section 4, we provide the summary and conclusion by making use of the result obtained in previous sections.

II. QUANTIZATION OF THE UNIT CELL EQUIVALENT CIRCUIT

The fundamental features of the LH TL of Fig 1.(a) are straightforwardly derived by elementary TL theory. It consists of loss-less per-unit equivalent circuit models (Fig 1.(b)) of the series-C/shunt-L prototype associated with NRI. The per-unit-length inductance L_l (H· m) and capacitance C_l (F· m) are $L_l = L_l' \cdot \Delta_z$ and $C_l = C_l' \cdot \Delta_z$, respectively. So we have the impedances $Z = 1/i\omega C_l$ (Ω/m) and admittances $Y = 1/i\omega L_l$ (S/m). According to the Fig.1(b), the complex propagation constant γ , the propagation constant β, the characteristic impedance Z_l, the phase velocity v_p, and the group velocity v_g of the unit cell equivalent circuit model for LH TL are given by[30]

$$\begin{aligned}
&\gamma = i\beta = \sqrt{ZY} = \frac{1}{i\omega\sqrt{C_l L_l}} = -i\frac{1}{\omega\sqrt{C_l L_l}},\\
&\beta = -\frac{1}{\omega\sqrt{C_l L_l}} < 0,\\
&Z_l = \sqrt{\frac{L_l}{C_l}},\\
&v_p = \frac{\omega}{\beta} = -\omega^2\sqrt{C_l L_l} < 0,\\
&v_g = (\frac{\partial\beta}{\partial\omega})^{-1} = \omega^2\sqrt{C_l L_l} > 0,
\end{aligned} \tag{1}$$

And the equivalent constitutive parameters for the unit cell equivalent circuit model in Fig.1(b) are[30]

$$\mu(\omega) = -\frac{1}{\omega^2 C_l} \tag{2}$$

$$\epsilon(\omega) = -\frac{1}{\omega^2 L_l} \tag{3}$$

According to Kirchhoff's law, the classical differential equations of motion of Fig.1(b) are

$$\begin{aligned}
\frac{du(z)}{dz} + \frac{j(z)}{i\omega C_l} = 0\\
\frac{dj(z)}{dz} + \frac{u(z)}{i\omega L_l} = 0
\end{aligned}$$

where u and j are the position-dependent voltage and currents $u = u(z)$ and $j = j(z)$ along the line, respectively. Then their corresponding second order partial differential equations are obtained as follows,

$$\frac{d^2u(z)}{dz^2} = \frac{1}{-\omega^2 C_l L_l}u(z) = -\gamma^2 u(z) \tag{4}$$

$$\frac{d^2j(z)}{dz^2} = \frac{1}{-\omega^2 C_l L_l}j(z) = -\gamma^2 j(z) \tag{5}$$

where γ is the complex propagation constant. And the walking-wave solutions to Eq(4) and Eq(5) reaches as

$$\begin{aligned}
u(z) = A\exp(-i\gamma z) + A^*\exp(i\gamma z)\\
j(z) = B\exp(-i\gamma z) + B^*\exp(i\gamma z)
\end{aligned}$$

in which A^* (B^*) are the conjugate complexes of A (B). In order to exploit quantum effects of mesoscopic LH TL equivalent circuit model, we adopt the quantization method similar to Louisell [31]. In the given unit-length, i.e.,$z_0 = m\lambda$ of Fig.1(b), where λ is the wavelength labelled typically by wavenumber k and frequency ω, the Hamiltonian can be written via the characteristic impedance relation,

$$\begin{aligned}
H &= \frac{1}{2}\int_0^{z_0}(L_l j^2(z) + C_l u^2(z))dz \quad = \int_0^{z_0} L_l j^2(z)dz\\
&= 2L_l A^* A z_0
\end{aligned}$$

where

$$\begin{aligned}
A = a\sqrt{\frac{\hbar\omega}{2L_l z_0}},\\
A^* = a^*\sqrt{\frac{\hbar\omega}{2L_l z_0}}.
\end{aligned}$$

Assume that the following equation was established with the energy units $\hbar\omega$. According to the canonical quantization principle, we can quantize the system by operators

$\hat{q}$ and $\hat{p}$, which satisfy the commutation relation $[\hat{q},\hat{p}]= i\hbar$. Then we can define the annihilation and creation operator $\hat{a}$ and $\hat{a}^{+}$ by the relations

$$\hat{a} = \frac{1}{\sqrt{2\hbar\omega}}(\omega\hat{q} + i\hat{p}),$$
$$\hat{a}^{\dagger} = \frac{1}{\sqrt{2\hbar\omega}}(\omega\hat{q} - i\hat{p})$$

Thus the quantum Hamiltonian of Fig.1(b) can be rewritten as

$$\hat{H} = \hbar\omega\hat{a}^{\dagger}\hat{a} = \frac{1}{2}(\omega^2\hat{q}^2 + \hat{p}^2)$$

which is as a quantum harmonic oscillator by imposing the quantizing condition $[\hat{q},\hat{p}]= i\hbar$. Where the variables $\hat{q}$ stand for the electric charges instead of the conventional "coordinates", while their conjugation variables $\hat{p}$ represent the electric currents instead of the conventional "momenta". Thus the current in the ideal non-dissipative unit cell equivalent circuit model for the LH TL can be quantized as

$$\hat{j}(z) = \sqrt{\frac{\hbar}{4\pi m L_l^2 C_l}}[\hat{a}\exp(\frac{i}{\omega\sqrt{C_l L_l}}z) + \hat{a}^{\dagger}\exp(-\frac{i}{\omega\sqrt{C_l L_l}}z)] \tag{6}$$

In the similar way, the quantum voltage operator of unit cell equivalent circuit model for this LH TL can be obtained as,

$$\hat{u}(z) = \sqrt{\frac{\hbar}{4\pi m L_l C_l^2}}[\hat{a}\exp(\frac{i}{\omega\sqrt{C_l L_l}}z) + \hat{a}^{\dagger}\exp(-\frac{i}{\omega\sqrt{C_l L_l}}z)] \tag{7}$$

III. NRI IN THERMAL FOCK STATE

The thermal noise shouldn't be ignored when the LH TL operates at some temperature. In the following, we exploit NRI dependent of the quantum fluctuations. As for the equilibrium situation, the so-called thermo field dynamics (TFD) extends the usual quantum field theory to the one at finite temperature[32]. In TFD, the tilde space accompanying with the Hilbert space, and the states and operators in the Hilbert space will find the corresponding objects in the tilde space. In this direct product space the degrees of freedom are double as the Hilbert space. The thermal degrees of freedom are introduced by doubling the degrees of freedom through tilde conjugation, and the tilde operators commute with the non-tilde operators in the tilde space[33]. Thus the creation and annihilation operators $\hat{a}^{\dagger}$, $\hat{a}$ associate with their tilde operators $\tilde{\hat{a}}^{\dagger}$, $\tilde{\hat{a}}$ according the rules:

$$[\tilde{\hat{a}}, \tilde{\hat{a}}^{\dagger}] = 1, \tag{8}$$
$$[\tilde{\hat{a}}, \hat{a}] = [\tilde{\hat{a}}, \hat{a}^{\dagger}] = [\hat{a}, \tilde{\hat{a}}^{\dagger}] = 0 \tag{9}$$

The number operators in the Hilbert space and tilde space are read as,

$$\hat{n} = \hat{a}^{\dagger}\hat{a},$$
$$\tilde{\hat{n}} = \tilde{\hat{a}}^{\dagger}\tilde{\hat{a}},$$

In the direct product space the thermal Fock state at finite temperature can be built by the thermal Bogoliubov transformation[33] through the Fock state $|\hat{n}\tilde{\hat{n}}\rangle = |\hat{n}\rangle \otimes |\tilde{\hat{n}}\rangle$ at zero temperature:

$$|\hat{n}\tilde{\hat{n}}\rangle_T = \hat{T}(\beta)|\hat{n}\tilde{\hat{n}}\rangle$$

where $\hat{T}(\beta)$ is a thermal unitary operator which is defined as

$$\hat{T}(\beta) = exp[-\beta(\hat{a}\tilde{\hat{a}} - \hat{a}^{\dagger}\tilde{\hat{a}}^{\dagger})] \tag{10}$$

the parameter β is the thermal unitary operator relating the thermal photos n_0 in the thermal vacuum state: $sinh\beta = n_0$. The thermal photos n_0 and temperature T are ruled by the Boltzmann distribution

$$n_0 = [exp(\hbar\omega/k_B T) - 1]^{-1} \tag{11}$$

in which k_B is the Boltzmann constant. The bosonic operators in TFD can relate each other by the thermal Bogoliubov transformation as following,

$$\hat{T}^{\dagger}(\beta)\hat{a}\hat{T}(\beta) = \mu\hat{a} + \tau\tilde{\hat{a}}^{\dagger}, \tag{12}$$
$$\hat{T}^{\dagger}(\beta)\hat{a}^{\dagger}\hat{T}(\beta) = \mu\hat{a}^{\dagger} + \tau\tilde{\hat{a}} \tag{13}$$

where μ=$cosh\beta$, τ=$sinh\beta$. Then from Eqs.(12) and (13), the average value of the current and the voltage in thermal Fock state can be calculated as,

$$\overline{\hat{j}(z)} = {}_T\langle\hat{n}\tilde{\hat{n}}|\hat{j}(z)|\hat{n}\tilde{\hat{n}}\rangle_T = 0, \tag{14}$$

$$\overline{\hat{u}(z)} = {}_T\langle\hat{n}\tilde{\hat{n}}|\hat{u}(z)|\hat{n}\tilde{\hat{n}}\rangle_T = 0, \tag{15}$$

Then the quantum fluctuation of the current in the unit cell equivalent circuit model is

$$\begin{aligned}\overline{(\Delta\hat{j})^2} &= \overline{\hat{j}(z)^2} - \overline{\hat{j}(z)}^2 = \overline{\hat{j}(z)^2} \\ &= \frac{\hbar}{4\pi m L_l^2 C_l}(1 + 2n_0)(1 + 2n)\end{aligned} \tag{16}$$

In the similar way, the quantum fluctuation of the voltage in the unit cell equivalent circuit model for LH TL is calculated as

$$\overline{(\Delta\hat{u})^2} = \frac{\hbar}{4\pi m L_l C_l^2}(1+2n_0)(1+2n) \tag{17}$$

Substituting Eq.(11) into Eq.(16) we can obtain

$$1+2n_0 = coth(\frac{\hbar\omega}{2k_B T}) \tag{18}$$

Then combining the characteristic parameters of the unit cell equivalent circuit model for LH TL in express (1) $\sim$ (3), Eq.(16) and Eq.(18), its constitutive parameter, i.e., the refractive index in microwave band[**?**] can be written as follows,

$$n_r = -\frac{2z_0 Z_l \overline{(\Delta\hat{j})^2}}{\hbar\omega^3(1+2n)coth(\frac{\hbar\omega}{2k_B T})} \tag{19}$$

IV. RESULTS AND DISCUSSION

According Eq.(19), the refractive index of the unit cell equivalent circuit of LH TL is dependent of the temperature, frequency, the number of photos, the characteristic impedance, the unit-length of the unit cell equivalent circuit and the quantum fluctuation of the current. To understand these dependence intuitively, we plot the refractive index dependent of the thermal number of photons in Fig.2, the diverse temperatures in Fig.3 and the frequencies in Fig.4, respectively.

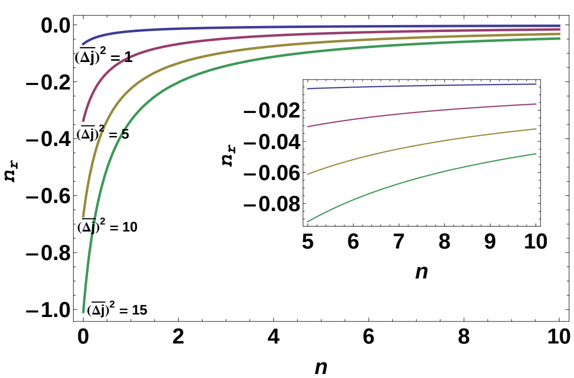

FIG. 2. (Color online) Dependence of the refractive index n_r and the thermal number of photons n under different quantum fluctuation of the current in the unit-length equivalent circuit model for LH TL.

The quantum fluctuation is the significant feature of the mesoscopic LH TL, and it may play an important role in the refractive index. Fig.2 shows the dependence of the refractive index n_r and the thermal number of photons n under different quantum fluctuation of the current with the parameters of $Z_l = 1\Omega/m$, $\omega = 2\pi f$, $f = 100GHz$(in microwave band), $T = 273K$. The curves in Fig.2 display the NRI get a larger values with less thermal photons. And at the weaker thermal photons level, the NRI values increase with the quantum fluctuation of the current. However, when the thermal photons $n > 5$ shown inset part in Fig.2, the increasing quantum fluctuation of the current influences NRI triflingly.

When the LH TL operates for a long time, the temperature T will be an important external conditional parameter for the NRI. In Fig.3, the characteristic impedance can be set $Z_l = 50\Omega/m$ with the current fluctuation $\overline{(\Delta\hat{j})^2} = 5$. The destructive dependence of the temperature T and the refractive index n_r is shown when the thermal photons vary from the low to moderate level. Fig.3 demonstrates the mesoscopic LH TL equivalent circuit should operate at low temperature T with low level thermal photons.

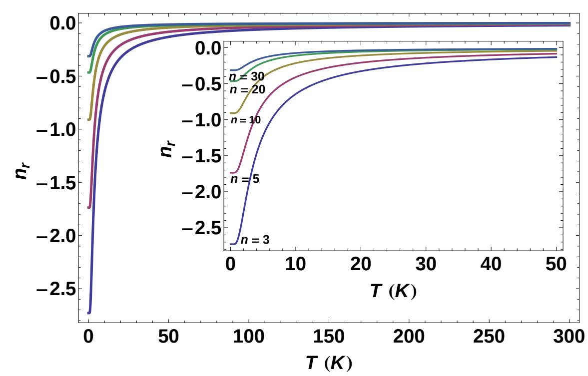

FIG. 3. (Color online) Dependence of the refractive index n_r and the temperature T under different thermal photons in the unit-length equivalent circuit model for LH TL.

On account of its refractive index of the LH TL being negative within the microwave frequency band, the frequency of electromagnetic wave propagating in the mesoscopic LH TL is another important parameter. Fig.4 shows NRI of the mesoscopic LH TL within the microwave frequency band [20] when it operates at room temperature $T = 300K$ with the quantum fluctuation of the current $\overline{(\Delta\hat{j})^2}$=18 and $Z_l = 50\Omega/m$. The curves show that NRI varies in the domain [0,-3] during the course of thermal photons increasing from 10 to 50 when the mesoscopic LH TL operates in [10,30]GHz, which shows the mesoscopic LH TL achieves NRI at a lower frequency within the microwave frequency band, as coincides with the macroscopic LH TL[34].

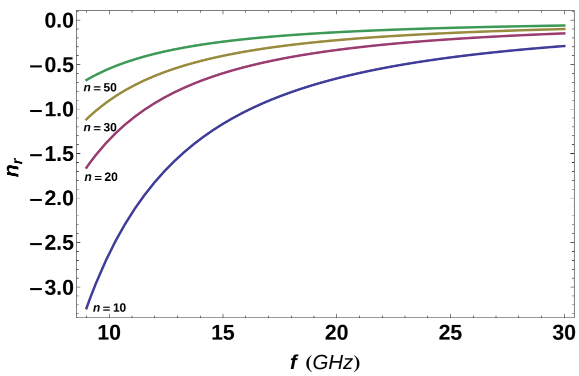

FIG. 4. (Color online) Dependence of the refractive index n_r and the frequency f under different thermal photons in the unit-length equivalent circuit model for LH TL.

Before concluding this paper, We also would like to point out that the dissipative mesoscopic LH TL equivalent circuit is not considered in this paper. Detailed investigation to the dissipative mesoscopic LH TL equivalent circuit will be presented in our forthcoming paper.

CONCLUSION

We proposed a quantized version for the loss-less mesoscopic LH TL circuit in thermal Fock by the TFD theory. With the thermal fluctuation of current we discussed NRI of the LH TL circuit. Under the fixed temperature, NRI is linear dependent of the thermal fluctuation of current and destructive dependent of the frequency within the microwave frequency band and the thermal photons. However, when the operating temperature increases the NRI is decreasing. Operating at a lower frequency within the microwave frequency band, temperature and little thermal photons, the mesoscopic LH TL is accessible to get NRI, which coincides with the macroscopic LH TL. And these results are significant for the the miniaturization of LH TL.

ACKNOWLEDGMENTS

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